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                ON DECOMPOSITIONS OF A MULTI-GRAPH INTO SPANNING SUBGRAPHS,(U)
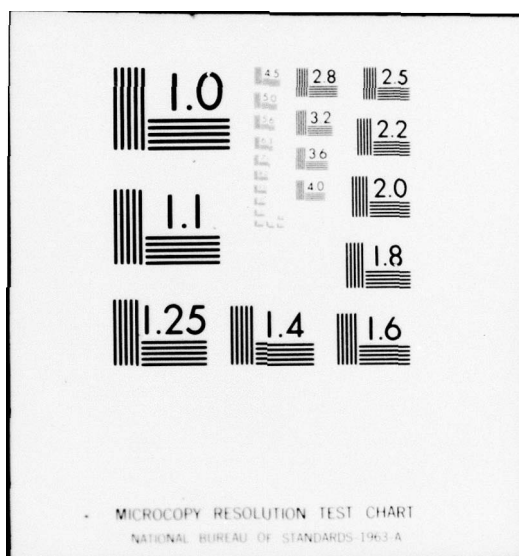   AUG 73   R P GUPTA   N00014-67-A-0232-0016

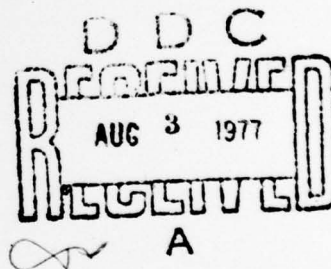UNCLASSIFIED   NL

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(6) ON DECOMPOSITIONS OF A MULTI-GRAPH

INTO SPANNING SUBGRAPHS,

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1. Let G be a <u>multi-graph</u>, i.e., a finite graph with no loops.
$V(G)$ and $E(G)$ denote the <u>vertex-set</u> and <u>edge-set</u> of G, respectively.
For $x \in V(G)$, $d(x, G)$ denotes the <u>degree</u> (or <u>valency</u>) of x in G
and $m(x, G)$ denotes the <u>multiplicity</u> of edges at x in G, i.e. the
minimum number m such that x is joined to any other vertex in G
by at most m edges.

A graph H is called a <u>spanning subgraph</u> of G if $V(H) = V(G)$
and $E(H) \subseteq E(G)$. Let k be any positive integer. Let

(1.1) $\sigma : G = H_1 \cup H_2 \cup \dots \cup H_k$

be a <u>decomposition</u> of G into k spanning subgraphs so that (1)
$H_1, H_2, \dots, H_k$ are spanning subgraphs of G, (2) $H_1, H_2, \dots, H_k$ are
pairwise edge-disjoint, and (3) $\bigcup_{1 \leq \alpha \leq k} E(H_\alpha) = E(G)$. For each
$x \in V(G)$, let $\nu(x, \sigma)$ denote the number of subgraphs H_α in σ
such that $d(x, H_\alpha) \geq 1$. Evidently,

(1.2) $\nu(x, \sigma) \leq \min \{k, d(x, G)\}$ for all $x \in V(G)$.

2. Given a multi-graph G and any positive integer k, we
consider the problem of determining a decomposition σ of G into k
spanning subgraphs such that $\nu(x, \sigma)$ is as large as possible for each
vertex $x \in V(G)$. In particular, we have proved the following two theorems.

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Theorem 2.1: *If* G *is a bipartite graph, then, for every positive integer* k, *there exists a decomposition* σ *of* G *into* k *spanning subgraphs such that*

$$(2.1) \qquad \nu(x, \sigma) = \min\{k, d(x, G)\} \text{ for all } x \in V(G).$$

Theorem 2.2: *If* G *is a multi-graph, then for every positive integer* k, *there exists a decomposition* σ *of* G *into* k *spanning subgraphs such that*

$$(2.2) \quad \nu(x, \sigma) \geq \begin{array}{l} \underline{\min}\{k - m(x, G), d(x, G)\} \text{ if } d(x, G) \leq k \\ \underline{\min}\{k, d(x, G) - m(x, G)\} \text{ if } d(x, G) \geq k \\ \qquad \text{for all } x \in V(G). \end{array}$$

Moreover, if $W \subseteq V(G)$ *is such that*

$$W \cap \{x \in V(G): \ k - m(x, G) < d(x, G) < k + m(x, G)\}$$

is independent, then σ *can be so chosen that in addition to* (2.2), *we have*

$$\nu(x, \sigma) = \min\{k, d(x, G)\} \text{ for all } x \in W.$$

3. The above theorems generalize some well-known theorems in graph theory.

Let G be a multi-graph; let H be a spanning subgraph of G. H is said to be a _matching_ of G if for every vertex x, $d(x, H) \leq 1$; H is said to be a cover of G if for every vertex x, $d(x, H) \geq 1$.

The chromatic index of G, denoted by $\chi_1(G)$, is defined to be the minimum number k such that there exists a decomposition of G into k spanning subgraphs each of which is a matching of G. The cover index of G, denoted by $\kappa_1(G)$ is the maximum number k such that there exists a decomposition of G into k spanning subgraphs each of which is a cover of G.

Theorems 3.1 and 3.2 below are obtained from Theorem 2.1 by taking $k = \max\limits_{x \in V(G)} d(x, G)$ and $k = \min\limits_{x \in V(G)} d(x, G)$, respectively.

Theorem 3.1 [1]: If G is a bipartite graph, then,

$$\chi_1(G) = \max\limits_{x \in V(G)} d(x, G).$$

Theorem 3.2 [2]: If G is a bipartite graph, then,

$$\kappa_1(G) = \min\limits_{x \in V(G)} d(x, G).$$

Similarly, Theorems 3.3 and 3.4 are seen to be special cases of Theorem 2.2.

Theorem 3.3 [3,4]: If G is a multi-graph, then,

$$\chi_1(G) \leq \max\limits_{x \in V(G)} \{d(x, G) + m(x, G)\}.$$

Theorem 3.4 [5]: If G is a multi-graph, then,

$$\kappa_1(G) \geq \min\limits_{x \in V(G)} \{d(x, G) - m(x, G)\}.$$

Remark: We have also generalized Theorem 2.1 to a theorem for balanced hypergraphs which contains as special cases some theorems due to C. Berge [6].

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